

A CATALOG OF COLOR-BASED REDSHIFT ESTIMATES FOR $z \lesssim 4$ GALAXIES IN THE HUBBLE DEEP FIELD

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ABSTRACT

We derive simple empirical color-redshift relations for $z \lesssim 4$ galaxies in the Hubble Deep Field (HDF) using a linear function of three photometric colors ($U-B$, $B-V$, $V-I$). The dispersion between the estimated redshifts and the spectroscopically observed ones is small for relations derived in several separate color regimes; the dispersions range from $\sigma_z \simeq 0.03$ to 0.1 for $z \lesssim 2$ galaxies, and from $\sigma_z \simeq 0.14$ to 0.25 for $z \gtrsim 2$ galaxies. We apply the color-redshift relations to the HDF photometric catalog and obtain estimated redshifts that are consistent with those derived from spectral template fitting methods. The advantage of these color-redshift relations is that they are simple and easy to use and do not depend on the assumption of any particular spectral templates; they provide model independent redshift estimates for $z \lesssim 4$ galaxies using only multiband photometry, and they apply to about 90% of all galaxies. We provide a color-based estimated redshift catalog of HDF galaxies to $z \lesssim 4$. We use the estimated redshifts to investigate the redshift distribution of galaxies in the HDF; we find peaks in the redshift distribution that suggest large-scale clustering of galaxies to at least $z \sim 1$ and that are consistent with those identified in spectroscopic probes of the HDF.

Key words: galaxies: distances and redshifts — methods: data analysis

1. INTRODUCTION

The Hubble Deep Field (HDF) provides accurate multiband photometry of galaxies to very faint magnitudes, with 10σ AB magnitude limit of $m_{814} = 27.60$ (Williams et al. 1996). The faint limit of the HDF makes it difficult to obtain spectroscopic redshifts for the majority of the galaxies in the field. It is therefore useful to derive estimated redshifts of these galaxies using the available multiband photometry. Several groups (Lanzetta, Yahil, & Fernández-Soto 1996; Gwyn & Hartwick 1996; Sawicki, Lin, & Yee 1997) obtained photometric redshifts for the HDF by comparing the observed $UBVI$ fluxes of each object with a set of galaxy spectral templates of different galaxy types redshifted to evenly spaced redshifts. Since spectroscopic redshifts have been measured and published for ~ 100 galaxies in the HDF (Cohen et al. 1996; Hogg et al. 1998; Steidel et al. 1996; Lowenthal et al. 1997), it is possible to fit analytic expressions for photometric redshifts. In this paper, we explore a simple empirical approach to estimating redshifts of galaxies based on their colors (see Connolly et al. 1997, Brunner et al. 1997, and Connolly et al. 1995 for an alternative empirical approach); this method has the advantage of being simple, model independent (i.e., it does not depend on the assumption of any particular set of galaxy spectral templates), and easy to use in determining approximate redshifts of $z \lesssim 4$ galaxies.

We determine the empirical analytic relations for color redshifts in § 2. We compare our estimated redshifts for the HDF galaxies with those obtained by the template-fitting method in § 3. We describe our estimated redshift catalog of HDF galaxies to $z \lesssim 4$ in § 4 (the Web site address of the catalog is given). We investigate the redshift clustering of HDF galaxies in § 5, and summarize our results in § 6.

2. EMPIRICAL COLOR-REDSHIFT RELATIONS

The HDF covers a 4 arcmin^2 area of sky in the northern continuous viewing zone (Williams et al. 1996). The HDF

galaxies have been observed in four passbands: m_{300} (which we denote by U), m_{450} (B), m_{606} (V), and m_{814} (I). We use the HDF photometric catalog by Sawicki et al. (1997); it contains 848 galaxies with measured fluxes in all four passbands to a magnitude limit of $I = 27$. Note that we use the AB system following Sawicki et al. (1997).

In deriving the color-based estimated redshift relations for the HDF, we use 82 galaxies with measured spectroscopic redshifts in this field, with redshifts in the range $z \simeq 0.1\text{--}3.5$ (Cohen et al. 1996; Hogg et al. 1998; Steidel et al. 1996; Lowenthal et al. 1997). Five of the galaxies (with $2.8 \leq z \leq 3.5$) are used with only upper limits to their U fluxes (as derived by Sawicki et al. 1997).

We first divide the galaxy sample into regions of high and low redshifts ($z \gtrsim 2$ and $z < 2$) based on empirical color cuts as follows. For $z \gtrsim 2$, the galaxies satisfy one of the following three color selection criteria:

$$U \geq 25.66, \quad U-B \geq 0.91, \quad B-V \leq 1.37, \quad V-I \leq 0.5; \quad (1)$$

$$I > 23.5, \quad U-B > 2.2; \quad (2)$$

$$I > 23.5, \quad B-V > 2.2, \quad U-B > -0.5. \quad (3)$$

Lower redshift galaxies, $z < 2$, generally fall outside these color-magnitude regions. Note that the above color selection criteria for $z \gtrsim 2$ galaxies reflect our current knowledge from galaxies with measured spectroscopic redshifts; they may need to be revised as new data become available. To minimize the redshift dispersion in the empirical color-redshift analytic fits, we further divide the regions into two color ranges for $z \gtrsim 2$ and three color ranges for $z < 2$. These empirical divisions reflect the color shifts as a function of redshift for different spectral type galaxies. For each color range, we fit an analytic relation for the redshift, z_a , which is linear in color,

$$z_a = c_1 + c_2(U-B) + c_3(B-V) + c_4(V-I), \quad (4)$$

where c_i ($i = 1, 4$) are constants. The error in z_a due to photometric errors is

$$\Delta z_a = [(c_2 \Delta U)^2 + |(c_3 - c_2)\Delta B|^2 + |(c_4 - c_3)\Delta V|^2 + (c_4 \Delta I)^2]^{1/2}. \quad (5)$$

We use equation (4) to determine the best fit between the observed spectroscopic redshifts of 82 HDF galaxies at $z \simeq 0.1$ –3.5 and their colors, in each of the five separate color ranges. We find the following best-fit relations and their redshift dispersions, σ_z . The redshift dispersions are calculated using the jack knife method (see Lupton 1993 for a simple description), which is also used to estimate the uncertainties of the coefficients in the best-fit relations. For easy reference, we assign each color range a number, cr , $1 \leq cr \leq 5$.

For the $z < 2$ galaxies (see photometric range discussed above):

1. $cr = 1: (U-B) < (B-V) - 0.1$: (28 galaxies)

$$z_a = 0.4111 - 0.1852(U-B) - 0.3062(B-V) + 0.7301(V-I), \quad \sigma_z = 0.034. \quad (6)$$

The 1σ uncertainties of the coefficients are 0.0036, 0.0058, 0.0084, and 0.0092, respectively.

2. $cr = 2: (U-B) \geq (B-V) - 0.1 > (V-I)$: (21 galaxies)

$$z_a = 0.163 - 0.171(U-B) + 0.340(B-V) + 0.194(V-I), \quad \sigma_z = 0.095. \quad (7)$$

The 1σ uncertainties of the coefficients are 0.013, 0.014, 0.045, and 0.047, respectively.

3. $cr = 3: (U-B) \geq (B-V) - 0.1 \leq (V-I)$: (19 galaxies)

$$z_a = 1.126 + 0.480(U-B) - 0.513(B-V) - 0.250(V-I), \quad \sigma_z = 0.097. \quad (8)$$

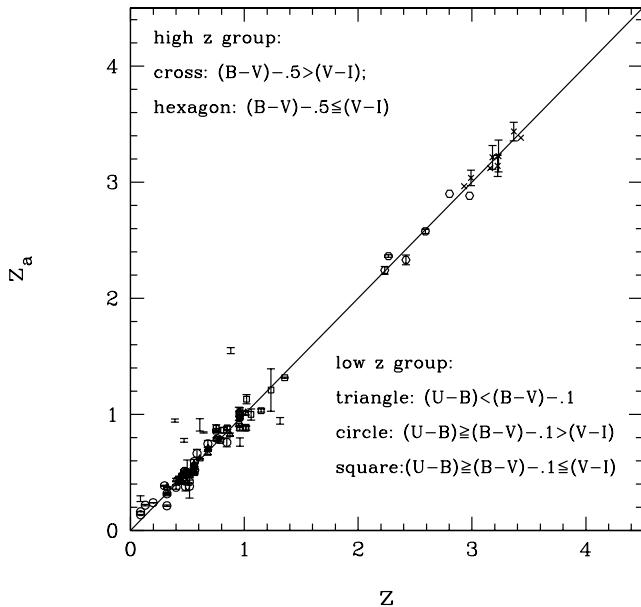


FIG. 1.—Color-based analytic redshift estimate z_a (given by eqs. [6]–[10]) vs. the spectroscopic redshift z for 90 HDF galaxies. The galaxies with known measurement errors in $UBVI$ are plotted with error bars in z_a .

TABLE 1

TABLE OF GALAXIES NOT USED IN DETERMINING THE z_a RELATIONS

ID Number	x	y	z	z_a	z_{temp}
20288.....	1831.00	753.00	0.609	0.9100	0.65
20456.....	527.00	1096.00	0.390	0.9474	1.10
30096.....	716.91	451.61	0.089	0.2757	2.00
30111.....	1463.00	489.00	1.315	0.9444	1.05
30251.....	1040.70	774.70	0.642	0.8456	0.70
30802.....	1618.80	1926.50	0.472	0.7770	0.85
40224.....	1961.00	599.00	0.961	0.7631	0.95
40818.....	1002.50	1771.70	0.882	1.5494	2.25

NOTES.—ID number is from the Sawicki et al. catalog; first number is chip number. x and y are pixel positions on v2 HDF images; z is spectroscopic redshift; z_a is color-based redshift; z_{temp} is redshift from spectral template fitting.

The 1σ uncertainties of the coefficients are 0.029, 0.023, 0.033, and 0.041, respectively.

For the $z \gtrsim 2$ galaxies (see photometric range above, eqs. [1]–[3]):

1. $cr = 4: (B-V) - 0.5 > (V-I)$: (eight galaxies; $z \gtrsim 3$)

$$z_a = 2.37 + 0.02(U-B) + 1.61(B-V) - 2.47(V-I), \quad \sigma_z = 0.14. \quad (9)$$

The 1σ uncertainties of the coefficients are 0.16, 0.04, 0.23, and 0.40, respectively.

2. $cr = 5: (B-V) - 0.5 \leq (V-I)$: (six galaxies; $z \lesssim 3$)

$$z_a = 2.18 + 0.10(U-B) + 0.20(B-V) + 0.75(V-I), \quad \sigma_z = 0.25. \quad (10)$$

The 1σ uncertainties of the coefficients are 0.14, 0.09, 0.34, and 0.77, respectively.

We present in Figure 1 the best-fit analytic redshifts z_a (from eqs. [6]–[10]) versus the measured spectroscopic galaxy redshifts z . The galaxies with known measurement errors in $UBVI$ are plotted with error bars in z_a . We used 82 out of 90 galaxies with available spectroscopic redshifts;¹ the eight error bars without points in Figure 1 denote the galaxies not used in determining the z_a relations (see Table 1). These eight outlying galaxies have a mean dispersion of $\sigma_z \simeq 0.45$ between their estimated redshifts z_a and their spectroscopic redshifts z ; they mostly lie near the boundaries of the five color ranges.

It is apparent from Figure 1 that the above simple relations provide good estimates of the galaxy redshifts. The constant offsets in equations (6)–(10) represent a rough indicator of the mean galaxy redshift in a given color range; the larger offsets represent higher redshift galaxies. The linear color-redshift relation used above is considerably simpler than the higher order polynomial fit used by Connolly et al. (1997) and has a smaller number of free parameters; it also yields a smaller dispersion ($\sigma_z \simeq 0.034$ vs. 0.097) for one of the $z \lesssim 2$ color ranges (see eq. [6]).

¹ We do not count the three $z > 2$ galaxies for which the published spectroscopic redshifts are erroneous or very uncertain (M. Sawicki 1998, private communication).

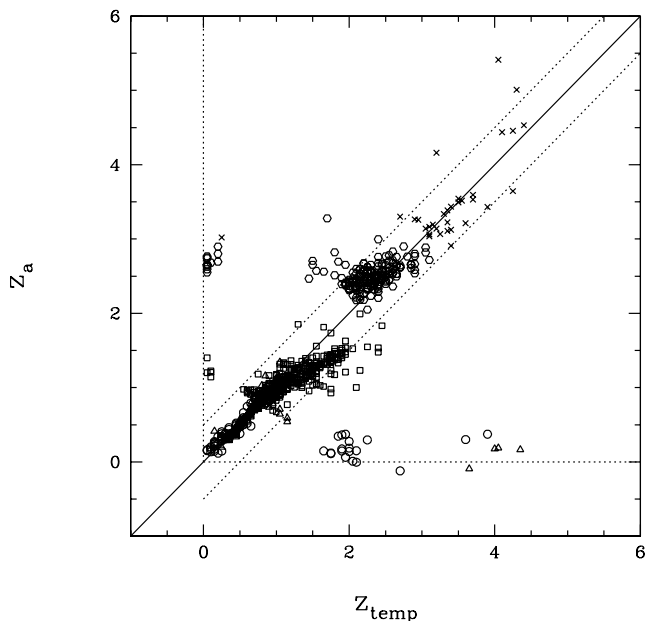


FIG. 2.—Color-based analytic redshift estimate z_a (given by eqs. [6]–[10]) vs. the Sawicki et al. (1997) template-fitting photometric redshift z_{temp} , for 848 galaxies in the HDF with $I < 27$ and measured $UBVI$. The symbols are the same as in Fig. 1. The solid diagonal line indicates $z_a = z_{\text{temp}}$; the dotted lines mark the region $|z_a - z_{\text{temp}}| \leq 0.5$.

In applying our formulae to the HDF photometric catalog, we first use equations (1)–(3) to select $z \gtrsim 2$ galaxies. In estimating color redshifts for $z < 2$ galaxies we then use equations (6)–(8), and for $z \gtrsim 2$ redshifts we use equations (9)–(10). In the next section, we compare these estimated color redshifts with those obtained from spectral template fitting.

3. COMPARISON WITH THE TEMPLATE-FITTING METHOD

Several groups (see, e.g., Lanzetta et al. 1996; Gwyn & Hartwick 1996; Sawicki et al. 1997) have obtained photometric redshifts for the HDF galaxies by comparing the observed $UBVI$ magnitudes with a set of galaxy templates of different spectral types redshifted to evenly spaced redshifts. The photometric redshifts obtained by these groups are generally consistent with each other, although large differences exist for some galaxies.²

The color-redshift relations derived in the present paper (§ 2) yield a considerably smaller dispersion between the estimated and spectroscopic redshifts than the template-fitting method; this occurs because our method explicitly minimizes the dispersion for each color range of galaxies with measured redshifts, which are used in fitting the color-redshift relations. A comparison of our predicted color redshifts, z_a , with the photometric redshifts from template-fitting by Sawicki et al. (1997), z_{temp} , is presented in Figure 2 for the 848 HDF galaxies with $I < 27$ and with measured $UBVI$ magnitudes. The solid diagonal line in Figure 2 indicates $z_a = z_{\text{temp}}$; the two dotted diagonal lines mark the region in which $|z_a - z_{\text{temp}}| \leq 0.5$. The results show that the two estimators, the simple color redshift estimator and the template-fitting redshift estimator, are generally consistent with each other, with some outliers. About 90% of all gal-

axies lie within $|z_a - z_{\text{temp}}| \leq 0.5$. From the 10% that lie outside this region (83 out of 848 galaxies), nearly half lie close to the region's boundary. The dozen discordant redshifts with $z_{\text{temp}} \sim 2$ and $z_a < 0.5$ (Fig. 2) are probably due to the gap in the available HDF spectroscopic redshifts at $z \sim 2$; thus galaxies with true redshifts of $z \sim 2$ may have been assigned wrong redshifts by the best-fit analytic formulae. The analytic formulae presented above can be improved as spectroscopic redshifts are measured for more galaxies in the HDF (especially in the missing redshift range of $1.4 \leq z \leq 2.2$).

The agreement between the analytic color redshifts we obtain and the spectral template-fitting photometric redshifts obtained by Sawicki et al. (1997) is comparable to the consistency among the various methods utilizing the spectral template-fitting technique. This illustrates that the linear analytic relations based on $UBVI$ colors provide a good and easy method for estimating galaxy redshifts. In fact, its dispersion in the different color ranges is lower than given by the template-fitting methods.

4. ESTIMATED REDSHIFT CATALOG OF HDF GALAXIES

We have calculated the estimated redshifts z_a of the HDF galaxies (848 galaxies with $I < 27$ and with measured $UBVI$ magnitudes) based on the color relations given above (§ 2). We present these redshifts in a catalog that is available by anonymous ftp in the `elt/HDF` subdirectory of `astro.princeton.edu`.

The estimated redshift catalog of HDF galaxies includes the following information for each galaxy: galaxy identification number; x and y pixel positions on the v2 HDF image; $UBVI$ magnitudes; our color redshift estimate, z_a (based on eq. [6]–[10]); photometric redshift from the template-fitting method by Sawicki et al., z_{temp} ; and, when available, the observed spectroscopic redshift, z . The color range number of each galaxy, cr (§ 2), is also listed in the catalog. Hard copies of the catalog are available upon request.

5. REDSHIFT CLUSTERING

We use the estimated color redshifts of the HDF galaxies to investigate the large-scale redshift clustering of galaxies. The small σ_z dispersion found for some of the color ranges provides the possibility of detecting large-scale clustering among the distant galaxies.

We use the estimated redshifts z_a for the 848 galaxies with $I < 27$ from the estimated redshift catalog of HDF galaxies (§ 4). We find that most of the galaxies satisfy either $(U - B) < (B - V) - 0.1$ (230 galaxies) or $(U - B) \geq (B - V) - 0.1 \leq V - I$ (333 galaxies). Since the number of galaxies is relatively large and the dispersion between z_a and z is relatively small [$\sigma_z = 0.034$ and $\sigma_{z_a} = 0.097$, respectively], we use these two groups to study the redshift distribution.

Figure 3 presents the estimated redshift distribution (using eq. [6]) of 230 HDF galaxies with $(U - B) < (B - V) - 0.1$ and $I < 27$, for a bin size of $\Delta z_a = 0.03$. The redshift distribution suggests the existence of peaks that indicate large-scale clustering of galaxies to $z \sim 1$. To estimate the significance of the observed structures in the redshift distribution, we calculate the expected smoothed distribution by convolving the observed redshift distribution with a Gaussian of width $\sigma'_z = 0.1$. The dashed and dotted lines in Figure 3 represent the mean smoothed

² Sawicki, M. J. 1997, <http://www.astro.utoronto.ca/~sawicki/>.

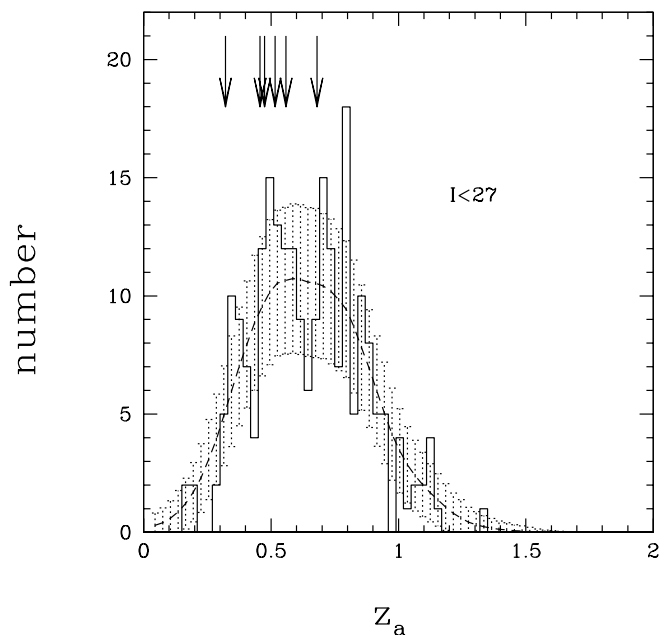


FIG. 3.—Estimated redshift (z_a) distribution of 230 galaxies with $(U-B) < (B-V) - 0.1$ and $I < 27$ in the HDF, for a bin size of $\Delta z_a = 0.03$. The dashed and the dotted lines represent the mean smoothed distribution and the 1σ contour, respectively, for $\sigma_z^r = 0.1$ and 10^4 realizations of the smoothed distribution (§ 5). The arrows on the top of the figure indicate the location of the peaks to $z \sim 0.8$ observed from spectroscopic redshifts of a smaller number of galaxies by Cohen et al. (1996).

distribution and the 1σ contour, respectively, for $\sigma_z^r = 0.1$ and 10^4 realizations of the smoothed distribution. Most of the observed peaks are marginally significant at levels of 1 to 3σ above the smoothed distribution. These peaks are consistent with the peaks revealed by spectroscopic observations of galaxies to $z \lesssim 0.8$ in this region (Cohen et al.

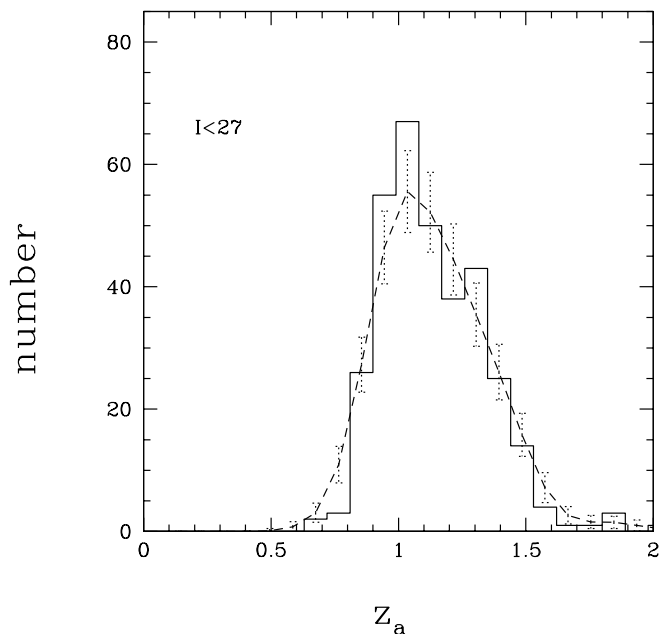


FIG. 4.—Estimated redshift (z_a) distribution of 333 galaxies with $(U-B) \geq (B-V) - 0.1 \leq V-I$ and $I < 27$ in the HDF, for a bin size of $\Delta z_a = 0.09$. The dashed and the dotted lines represent the mean smoothed distribution and the 1σ contour, respectively, for $\sigma_z^r = 0.09$ and 10^3 realizations of the smoothed distribution (§ 5).

1996); the location of the spectroscopic peaks are marked by the arrows on top of Figure 3. We see that the location of our suggested peaks are consistent with those seen directly with a smaller number of spectroscopically measured redshifts. Figure 3 suggests an additional peak at $z \sim 0.8$ that is not yet confirmed by spectroscopic data. The peaks are seen consistently in different parts of the HDF field, with no evidence of subclustering on the sky. These peaks suggest large-scale clustering in the galaxy distribution at high redshifts; they may represent the distant ($z \sim 1$) counterpart to local superclusters, or walls, seen at low redshifts (Broadhurst et al. 1990; Bahcall 1991). Most recently, such peaks have also been seen at $z \simeq 3$ (Steidel et al. 1998).

Figure 4 presents the redshift distribution of 333 HDF galaxies with $(U-B) \geq (B-V) - 0.1 \leq V-I$ and $I < 27$, for a bin size of $\Delta z_a = 0.09$. (For this group, the redshift dispersion is $\sigma_z = 0.097$). The dashed and the dotted lines in Figure 4 represent the mean smoothed distribution and the 1σ contour, respectively, for $\sigma_z^r = 0.09$ and 10^3 realizations of the smoothed distribution. A peak is suggested at $z_a \sim 1$ and possibly at $z \sim 1.3$, but the large dispersion for this color subsample appears to “wash out” any other significant underlying peaks.

6. SUMMARY AND DISCUSSION

Using HDF photometric and spectroscopic data, we have determined a set of simple analytic formulae that yield estimated galaxy redshifts to $z \lesssim 4$ in terms of linear combinations of three measured colors, $U-B$, $B-V$, and $V-I$ (eqs. [6]–[10]). The derived analytic formulae in five color ranges exhibit small dispersions between the estimated and spectroscopic redshifts. For $z \lesssim 2$ galaxies, the redshift dispersion ranges from $\sigma_z = 0.034$ to $\sigma_z = 0.097$ for different color ranges. For $z \gtrsim 2$ galaxies, we find $\sigma_z = 0.14$ and $\sigma_z = 0.36$ for two color ranges that typically represent $z \gtrsim 3$ and $z \lesssim 3$ galaxies, respectively. These color-redshift relations apply to about 90% of the galaxies in the sample.

The smallest dispersion between the color and the spectroscopic redshifts, $\sigma_z = 0.034$, occurs for the $z \lesssim 2$ galaxies satisfying $(U-B) < (B-V) - 0.1$; 28 galaxies with measured redshifts are used in deriving the relation for the estimated redshift, with only four free parameters (the coefficients in eq. [6]). There are 230 HDF galaxies with $I < 27$ and measured $UBVI$ magnitudes that belong to this color range; we investigate the large-scale redshift distribution of these galaxies and find evidence for peaks in the redshift distribution that suggest large-scale clustering to at least $z \sim 1$. These results are consistent with those of Cohen et al. (1996) using observed spectroscopic redshifts of a smaller number of galaxies.

We have applied our color redshift formulae to the entire HDF photometric catalog and find that the derived redshifts are consistent with those obtained from spectral template-fitting techniques. The analytic relations, by design, yield lower dispersion than the template-fitting method. The color-redshift relations have the advantage of being simple, model independent, and easy to use. They can be further improved with additional data. These analytic color-redshift estimators are useful in providing empirical estimates of galaxy redshifts to $z \lesssim 4$ using multiband photometry.

Our estimated redshift catalog of HDF galaxies, based on our color redshift formulae for all 848 HDF galaxies with $I < 27$ and measured $UBVI$ fluxes, is available by

anonymous ftp in the elt:/HDF subdirectory of astro.princeton.edu.

Note that our color-redshift relations (eqs. [6]–[10]) are derived using AB magnitudes and for the HDF filters. For application to other photometric catalogs, the appropriate spectroscopic training set should be used; when such a training set is not available, equations (6)–(10) may provide useful estimates after appropriate photometric transformation has been performed between the different filter systems. Also note that these color-redshift relations should not be applied to galaxies that lie close to the boundaries of the color ranges.

Finally, we note that our color-redshift relations are limited by the absence of measured spectroscopic redshifts for galaxies in the range of $1.4 \leq z \leq 2.2$ (see Figs. 1 and 2).

It is very important to obtain spectroscopic redshifts in this range, because it will not only enable better calibration of photometric redshifts, it will also help us understand the nature of galaxies in the intermediate redshift range.

It is a pleasure to thank Marcin Sawicki for providing the photometric catalog of the HDF and the corresponding photometric redshifts, as well as helpful comments on the manuscript; Daniela Calzetti for helpful communications concerning spectral templates; Judy Cohen and David Hogg, who also provided helpful comments on the manuscript, for the use of redshifts prior to publication; and James Rhoads for help with IRAF. This work is partially supported by NSF grants AST 93-15368 and AST 98-02802.

APPENDIX

Table 2 presents our catalog of color-based redshift estimates for $z \lesssim 4$ galaxies in the Hubble Deep Field.

TABLE 2
CATALOG OF COLOR-BASED REDSHIFT ESTIMATES

ID	x	y	m_{814}	m_{606}	m_{450}	m_{300}	z_{temp}	z_a	cr	z
20005.....	1438.00	137.00	26.69	26.95	26.86	28.77	2.20	2.56	5	
20007.....	372.24	151.40	26.69	26.75	26.72	27.32	1.75	1.41	3	
20009.....	1510.50	154.20	26.65	26.82	26.86	27.17	1.45	1.21	3	
20010.....	708.44	160.54	26.68	27.28	28.07	28.53	0.55	0.52	1	
20015.....	229.00	172.00	26.60	27.11	28.14	29.50	0.25	0.38	2	

NOTES.—Table 2 is presented in its entirety in the electronic edition of the *Astronomical Journal*. A portion is shown here for guidance regarding its form and content. “ID” is identification number in the Sawicki et al. photometric catalog; first digit is chip number. x and y are pixel positions on v2 HDF image; z_{temp} is photometric redshift from fitting spectral templates by Sawicki et al. 1997; z_a is color-based analytic redshift from eqs. (6)–(10); cr is color range; z is spectroscopic redshift. An asterisk indicates the 83 galaxies for which $|z_a - z_{\text{temp}}| > 0.5$; four of these have spectroscopic redshifts z , and $|z_a - z|$ is much smaller than $|z_{\text{temp}} - z|$; about half of the 83 galaxies lie close to the border of $|z_a - z_{\text{temp}}| \leq 0.5$. A question mark indicates the three galaxies for which the published spectroscopic redshifts are erroneous or very uncertain.

REFERENCES

- Bahcall, N. A. 1991, *ApJ*, 376, 43
 Broadhurst, T. J., Ellis, R. S., Koo, D. C., & Szalay, A. 1990, *Nature*, 343, 726
 Brunner, R. J., Connolly, A. J., Szalay, A. S., & Bershad, M. A. 1997, *ApJ*, 482, L21
 Cohen, J. G., Cowie, L. L., Hogg, D. W., Songaila, A., Blandford, R., Hu, E. M., & Shopbell, P. 1996, *ApJ*, 471, L5
 Connolly, A. J., Csabai, I., Szalay, A. S., Koo, D. C., Kron, R. C., & Munn, J. A. 1995, *AJ*, 110, 2655
 Connolly, A. J., Szalay, A. S., Dickinson, M., SubbaRao, M. U., & Brunner, R. J. 1997, *ApJ*, 486, L11
 Gwyn, S. D. J., & Hartwick, F. D. A. 1996, *ApJ*, 468, L77
 Hogg, D. W., et al. 1998, *AJ*, in press
 Lanzetta, K. M., Yahil, A., & Fernández-Soto, A. 1996, *Nature*, 381, 759
 Lowenthal, J. D., et al. 1997, *ApJ*, 481, 673
 Lupton, R. 1993, *Statistics in Theory and Practice* (Princeton: Princeton Univ. Press)
 Sawicki, M. J., Lin, H., & Yee, H. K. C. 1997, *AJ*, 113, 1
 Steidel, C. C., Adelberger, K., Dickinson, M., Giavalisco, M., Pettini, M., & Kellogg, M. 1998, *ApJ*, 492, 428
 Steidel, C. C., Giavalisco, M., Dickinson, M., & Adelberger, K. L. 1996, *ApJ*, 462, 17
 Williams, R. E., et al. 1996, *AJ*, 112, 1335